Conditionally Stable Amplifier Design Using Constant μ-Contours

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Abstract— **Conditionally stable amplifiers ($0 < k < 1$) can be an attractive design option for commercial applications where increased gain can be achieved by accepting a measured risk of instability. A procedure is presented for designing amplifier input and output matching networks which permits an a-priori, quantitative trade-off between gain and the stability parameter, μ (or μ'). Analytical results have been derived suitable for CAD implementation.**

I. Introduction

CONDITIONALLY STABLE amplifiers can be designed with either the input or output conjugately matched but not both [1], [2]. If the output is to be matched then the overall gain of the amplifier, referred to as the available gain [3], is a function of the source reflection coefficient, Γ_S. Edwards et. al. [2] have shown that a specific available gain circle, designated as the Maximum Single-sided Match (MSM) gain circle, delineates a region in the Smith Chart where Γ_S produces an output impedance that can be conjugately matched in the stable region of the load plane. The gain associated with the MSM circle, determines the maximum available gain achievable under these conditions. While the MSM circle shows the desirable region for Γ_S it does not indicate *quantitatively* the degree of stability or instability that the overall amplifier design will have. This is usually considered by examining the stability circles of the overall amplifier once the design is completed to see the extent to which they encroach the passive Smith Chart region. A quantitative implementation of this is achieved by calculating the stability parameters μ and μ' [4], [5] of the completed amplifier. It would be desirable to know the degree of stability implied by a particular choice of Γ_S prior to completing the amplifier design. To this end a method is developed to forecast the value of μ and μ' for the complete amplifier based on the Γ_S choice. It is not obvious that such a procedure is possible since there are multiple implementations of matching networks. However, this paper will show that for IMN (Input Matching Network) and OMN (Output Matching Network) which are *passive, lossless,* and *reciprocal* that μ and μ' of the complete amplifier are uniquely defined by Γ_S and the transistor S-parameters. Consequently, it is possible to plot contours of constant μ and μ' on the source Smith Chart which are shown to coincide with the constant available gain circles. The choice of Γ_S can therefore be made by considering the trade-off between gain and stability. In reality, actual matching networks will have some slight loss but this tends to improve stability. Hence this approach is conservative in that the measured stability is usually better than the designed stability. The procedure is illustrated with a practical example.

By duality, if the input of a conditionally stable amplifier is to be matched then the gain, referred to as operating power gain, is a function of the load reflection coefficient, Γ_L. Again if the IMN and OMN are passive, lossless and reciprocal networks then μ and μ' of the complete amplifier are determined by Γ_L thereby permitting the designer to make a trade off between gain and stability.

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II. Matching Circuit Properties

Matching circuit are most often created by combining passive elements such as inductors, capacitors, transmission lines, and open circuited or short circuited transmission line stubs. It is often a good approximation at RF and microwave frequencies to assume that these elements are lossless. Collin [6] has shown that the S-parameters for a Passive Lossless Reciprocal Network (PLRN) are of the form

$$S_{\mathrm{PLRN}} = \begin{pmatrix} S_{11} & \sqrt{1-|S_{11}|^2}e^{j\gamma} \\ \sqrt{1-|S_{11}|^2}e^{j\gamma} & -S_{11}e^{j2\gamma} \end{pmatrix}. \quad (1)$$

Therefore, an OMN that transforms Γ_L to 50 Ω has an S-matrix equal to

$$S_{\mathrm{OMN}} = \begin{pmatrix} \Gamma_L & \sqrt{1-|\Gamma_L|^2}e^{j\gamma} \\ \sqrt{1-|\Gamma_L|^2}e^{j\gamma} & -\Gamma_L^* e^{j2\gamma} \end{pmatrix}.$$

Similarly, an IMN that transforms 50 Ω to Γ_S has an S-matrix equal to

$$S_{\mathrm{IMN}} = \begin{pmatrix} -\Gamma_S^* e^{j2\gamma} & \sqrt{1-|\Gamma_S|^2}e^{j\gamma} \\ \sqrt{1-|\Gamma_S|^2}e^{j\gamma} & \Gamma_S \end{pmatrix}.$$

Obviously, the S-parameters of the matching network are determined by Γ_S (or Γ_L) and an arbitrary phase γ. Using signal flow graphs one can see that the phase angle, γ, equates to an arbitrary length of transmission line. In both the IMN and OMN, the transmission line is connected to the 50 Ω impedance side of the matching network. This is illustrated in Fig. 1.

Since a transmission line added to the input of an amplifier only rotates the source stability circle about the center of the Smith Chart, it follows that μ' *is invariant with respect to a transmission line* (Z_0) *added to the input.* This can be seen analytically by directly calculating μ' for a circuit with an

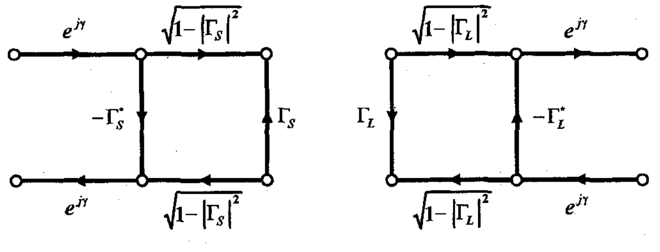

Fig. 1. A signal flow graph representation of an IMN and OMN showing that s-parameters are uniquely determined by Γ_S and Γ_L except for an arbitrary length of transmission line.

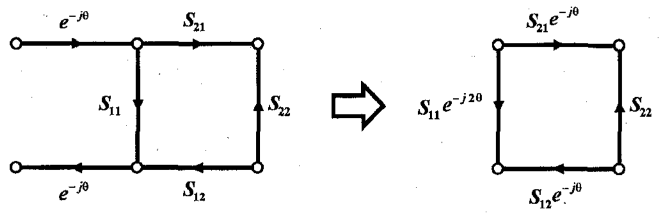

Fig. 2. Signal flow graphs showing the combination of a transmission line and a general two-port circuit.

added transmission line and observing that it reduces to μ' of the circuit alone. This is illustrated in Fig. 2 and the equations that follow:

$$\mu'(\mathrm{TLIN}+\boldsymbol{S}) = \frac{1-|S_{22}|^2}{|S_{11}e^{-j2\theta}-S_{22}^*\Delta_{\mathrm{TLIN}+\boldsymbol{S}}|+|S_{21}e^{-j\theta}S_{12}e^{-j\theta}|} = \frac{1-|S_{22}|^2}{|S_{11}-S_{22}^*\Delta_{\boldsymbol{S}}|+|S_{21}S_{12}|} = \mu'(\boldsymbol{S})$$

where $\Delta_{\boldsymbol{S}} = S_{11}S_{22} - S_{12}S_{21}$.

By duality μ is invariant with respect to a transmission line (Z_0) added to the output side

It can be shown by directly applying the S-parameters of a PLRN that it maps the USC (Unit Smith Chart) onto itself. In general

$$\Gamma_{\mathrm{OUT}} = \frac{S_{22}-\Delta_{\boldsymbol{S}}\Gamma_S}{1-S_{11}\Gamma_S} \tag{2}$$

and substitution from (1) indicates that $\Delta_{\boldsymbol{S}} = -e^{j2\gamma}$ and

$$\begin{aligned}
&|\Gamma_{\mathrm{OUT}}| \le 1\\
&\Leftrightarrow \left|\frac{-S_{11}^*e^{j2\gamma}+\Gamma_S e^{j2\gamma}}{1-S_{11}\Gamma_S}\right|^2 \le 1\\
&\Leftrightarrow (-S_{11}^*+\Gamma_S)(-S_{11}+\Gamma_S^*)\\
&\le (1-S_{11}\Gamma_S)(1-S_{11}^*\Gamma_S^*)\\
&\Leftrightarrow (1-|S_{11}|^2)|\Gamma_S|^2 \le (1-|S_{11}|^2)\\
&\Leftrightarrow |\Gamma_S| \le 1.
\end{aligned}$$

By symmetry, for a PLRN, $|\Gamma_{\mathrm{IN}}| \le 1 \Leftrightarrow |\Gamma_L| \le 1$.

Therefore, the source stability circle and $\mu'$are invariant with respect to the addition of a PLRN to the output as illustrated in Fig. 3 where T_{ij} denotes the transistor scattering parameters.

By duality the load stability circle and μ are invariant with respect to the addition of a PLRN to the input.

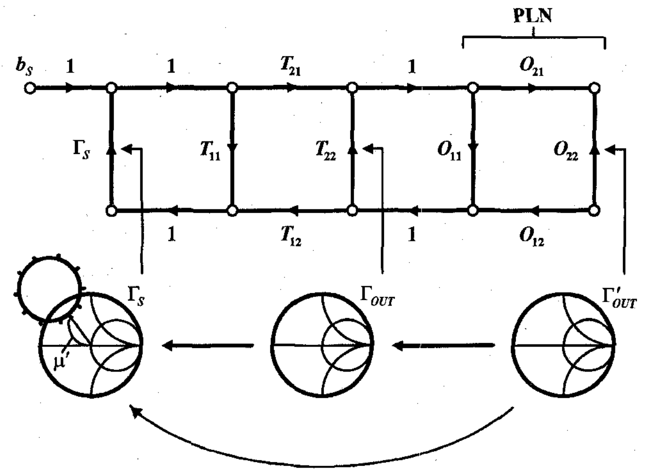

Fig. 3. Source stability circle and μ' are invariant with respect to the addition of a PLRN to the output.

III. Constant μ'-Contours in the Source Plane

In this section it is shown that μ' of the completed amplifier is determined by the transistor's scattering parameters, T_{ij}, and the source reflection coefficient, Γ_S, as viewed from the transistor. This follows because the S-parameters of the IMN are determined by Γ_S except for an arbitrary transmission line at the input. Since μ' is invariant with respect to a transmission line of characteristic impedance Z_0 at the input, the line length can be set to zero. And, because μ' is invariant to a passive and lossless OMN, it can be calculated by combining the S-parameters of the transistor with those of the IMN alone. This is illustrated in Fig. 4. The S-matrix for the combined IMN and transistor is therefore

$$U(\Gamma_S,\boldsymbol{T}) = \frac{1}{1-T_{11}\Gamma_S} \cdot \begin{pmatrix} T_{11}-\Gamma_S^* & T_{12}\sqrt{1-|\Gamma_S|^2} \\ T_{21}\sqrt{1-|\Gamma_S|^2} & T_{22}-\Delta_{\boldsymbol{T}}\Gamma_S \end{pmatrix}. \tag{3}$$

The stability parameter μ' of the amplifier is then

$$\mu' = \frac{1-|U_{22}|^2}{|U_{11}-\Delta_{\boldsymbol{U}}U_{22}^*|+|U_{12}U_{21}|}$$

which can be shown [4] to be equivalent to

$$\mu' = \frac{|U_{11}-\Delta_{\boldsymbol{U}}U_{22}^*|-|U_{12}U_{21}|}{|U_{11}|^2-|\Delta_{\boldsymbol{U}}|^2}. \tag{4}$$

Substitution of the matrix elements from (3) into (4) yields,

$$\mu' = \frac{|C_1-B_1\Gamma_S^*+C_1^*(\Gamma_S^*)^2|-|T_{12}T_{21}(1-|\Gamma_S|^2)|}{|T_{11}-\Gamma_S^*|^2-|\Delta_{\boldsymbol{T}}-T_{22}\Gamma_S^*|^2}. \tag{5}$$

Or

$$\mu' = \frac{|C_1-B_1\Gamma_S^*+C_1^*(\Gamma_S^*)^2|-|T_{12}T_{21}(1-|\Gamma_S|^2)|}{E_1|\Gamma_S|^2-C_1\Gamma_S-C_1^*\Gamma_S^*+D_1} \tag{6}$$

where

$$B_1 = D_1 + E_1 \tag{7a}$$
$$C_1 = T_{11} - \Delta_{\boldsymbol{T}}T_{22}^* \tag{7b}$$
$$D_1 = |T_{11}|^2 - |\Delta_{\boldsymbol{T}}|^2 \tag{7c}$$
$$E_1 = 1 - |T_{22}|^2. \tag{7d}$$

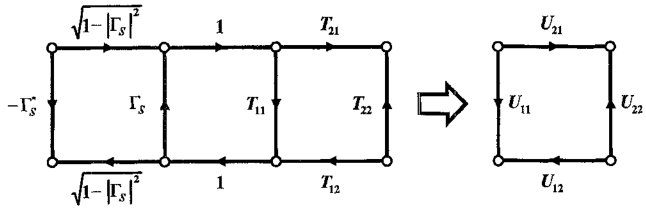

Fig. 4. μ' of the completed amplifier can be calculated using the scattering-matrix U which results from combining the IMN and transistor.

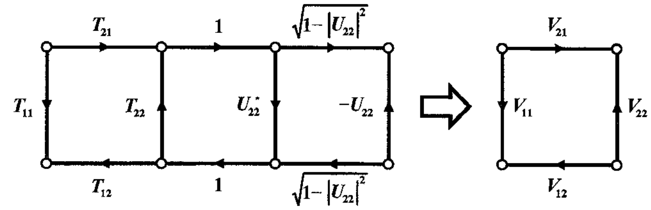

Fig. 5. μ of the completed amplifier can be calculated from the scattering-matrix V for the combined transistor and OMN circuits.

It is interesting to note that the first term with absolute sign in the numerator can be expressed in terms of the invariant points [2], [6] as below

$$|C_1 - B_1\Gamma_S^* + C_1^*(\Gamma_S^*)^2| = |C_1(\Gamma_S - \Gamma_S^+)(\Gamma_S - \Gamma_S^-)|$$

where Γ_S^+ and Γ_S^- are invariant points in the Γ_S plane, i.e., the points on the USC for which all constant gain circles intersect. The term inside the second absolute value sign in the numerator of (6) is always positive provided Γ_S is in the USC, i.e., $|\Gamma_S| \le 1$ and in that case the absolute value sign can be dropped. Therefore

$$\mu' = \frac{|C_1 - B_1\Gamma_S^* + C_1^*(\Gamma_S^*)^2| - |T_{12}T_{21}|(1 - |\Gamma_S|^2)}{E_1|\Gamma_S|^2 - C_1\Gamma_S - C_1^*\Gamma_S^* + D_1}.$$

Or

$$\mu'(E_1|\Gamma_S|^2 - C_1\Gamma_S - C_1^*\Gamma_S^* + D_1) + |T_{12}T_{21}|(1 - |\Gamma_S|^2) = |C_1 - B_1\Gamma_S^* + C_1^*(\Gamma_S^*)^2|. \quad (8)$$

Direct substitution shows that indeed the invariant points, $\Gamma_S = \Gamma_S^\pm$, satisfy (8) for all values of μ'. This follows since $C_1\Gamma_S^\pm + C_1^*\Gamma_S^{\pm^*} - B_1 = 0, |\Gamma_S^\pm| = 1$, and

$$E_1|\Gamma_S^\pm|^2 - C_1\Gamma_S^\pm - C_1^*\Gamma_S^{\pm^*} + D_1 = E_1 - B_1 + D_1 = 0.$$

By squaring (8) and using the fact that $|z|^2 = zz^*$ one can eliminate the absolute value sign from the right-hand-side to get the following complex variable equation in Γ_S.

$$\begin{aligned}
&[(\mu'E_1 - |T_{12}T_{21}|)^2 - |C_1|^2]|\Gamma_S|^4 \\
&\quad + C_1[B_1 - 2(\mu')^2E_1 + 2\mu'|T_{12}T_{21}|]\Gamma_S|\Gamma_S|^2 \\
&\quad + C_1^*[B_1 - 2(\mu')^2E_1 + 2\mu'|T_{12}T_{21}|]\Gamma_S^*|\Gamma_S|^2 \\
&\quad + [(\mu')^2 - 1]C_1^2\Gamma_S^2 + [(\mu')^2 - 1](C_1^*)^2(\Gamma_S^*)^2 \\
&\quad + [2(\mu'D_1 + |T_{12}T_{21}|)(\mu'E_1 - |T_{12}T_{21}|) \\
&\qquad + 2(\mu')^2|C_1|^2 - B_1^2]|\Gamma_S|^2 \\
&\quad + C_1[B_1 - 2(\mu')^2D_1 - 2\mu'|T_{12}T_{21}|]\Gamma_S \\
&\quad + C_1^*[B_1 - 2(\mu')^2D_1 - 2\mu'|T_{12}T_{21}|]\Gamma_S^* \\
&\quad + (\mu'D_1 + |T_{12}T_{21}|)^2 - |C_1|^2 = 0. \qquad (9)
\end{aligned}$$

Equation (9) which involves fourth powers of the complex variable Γ_S can be manipulated into the following form which in general defines two circles in the complex plane

$$(|\Gamma_S - M|^2 - P)(|\Gamma_S - N|^2 - Q) = 0$$

provided that the lead coefficient in (9) $(\mu'E_1 - |T_{12}T_{21}|)^2 - |C_1|^2 \neq 0$. In the case where this condition is not met, the Γ_S circles are observed to become straight lines passing through both invariant points. Straight lines are a limiting case of circles where radii and centers approach infinity.

The centers of the circles, M and N along with the square radii, P and Q are given below

$$M = \frac{|C_1|}{E_1}\hat{c}_S \quad (10a)$$

$$P = \frac{|T_{12}T_{21}|^2}{E_1^2} \quad (10b)$$

$$N = \frac{|C_1|[1 - (\mu')^2]}{E_1[1 - (\mu')^2] - 2|T_{12}T_{21}|(k - \mu')}\hat{c}_S \quad (11a)$$

$$Q = \frac{|T_{12}T_{21}|^2[(\mu')^2 - 2k\mu' + 1]^2}{\{E_1[1 - (\mu')^2] - 2|T_{12}T_{21}|(k - \mu')\}^2} \quad (11b)$$

and $\hat{c}_S = C_1^*/|C_1|$ [2] is the unit vector along the direction from the center of the USC to the center of the source stability circle.

A comparison of the expressions given by (10) and (11) shows that the results of (10) are included in those of (11) as a special case where $\mu' = k$. Thus the expressions in (11) represent the general solution. The constant μ' contour on the Γ_S plane is therefore a circle which intersects the invariant points and is represented by the following expression.

$$\left|\Gamma_S - \frac{|C_1|[1 - (\mu')^2]}{E_1[1 - (\mu')^2] - 2|T_{12}T_{21}|(k - \mu')}\hat{c}_S\right| = \left|\frac{(T_{12}T_{21})[(\mu')^2 - 2k\mu' + 1]}{E_1[1 - (\mu')^2] - 2|T_{12}T_{21}|(k - \mu')}\right|. \quad (12)$$

The available power gain, G_A, is defined in terms of the normalized gain, g_a, as $G_A = g_a|T_{21}|^2$ where

$$g_a = \frac{1 - |\Gamma_S|^2}{|1 - T_{11}\Gamma_S|^2 - |T_{22} - \Delta_T\Gamma_S|^2}.$$

It is well known that contours of constant gain [7] appear as circles on the Smith Chart with centers defined by

$$C_{ga} = \frac{|C_1|}{D_1 + \frac{1}{g_a}}\hat{c}_S. \quad (13)$$

The centers of the constant gain circles and the center of the constant μ' circles lie on a common ray drawn from the center of the Smith Chart defined by the unit vector $\hat{c}_S$. Also, it is known that the constant μ' contour passes through the invariant points. Therefore, the constant μ' circles are the same family of circles as the available gain circles. For a particular constant μ' circle, it's corresponding g_a value can be found by equating

TABLE I
FORMULAS FOR CONSTANT μ' AND μ CONTOURS IN THE Γ_S-PLANE FOR THE OUTPUT CONJUGATELY MATCHED CONDITIONALLY STABLE AMPLIFIER

Output Conjugately Matched Conditionally Stable Amplifier		
Γ_s-Plane	Constant μ'-Contour	Constant μ-Contour
Center	$\frac{\lvert C_1\rvert[1-(\mu')^2]}{E_1[1-(\mu')^2]-2\lvert T_{12}T_{21}\rvert(k-\mu')}\hat{c}_S$	$\frac{2\lvert C_1\rvert(k-m)}{\lvert T_{12}T_{21}\rvert(1-m^2)+2D_1(k-m)}\hat{c}_S$
Radius	$\left\lvert\frac{(T_{12}T_{21})[(\mu')^2-2k\mu'+1]}{E_1[1-(\mu')^2]-2\lvert T_{12}T_{21}\rvert(k-\mu')}\right\rvert$	$\left\lvert\frac{(T_{12}T_{21})(\mu^2-2k\mu+1)}{\lvert T_{12}T_{21}\rvert(1-\mu^2)+2D_1(k-\mu)}\right\rvert$
Available Gain, G_A	$\frac{[1-(\mu')^2]}{2\mu'(1-k\mu')}\cdot MSG$	$\frac{2(k-\mu)}{(1-\mu^2)}\cdot MSG$

TABLE II
FORMULAS FOR CONSTANT μ AND μ' CONTOURS IN THE Γ_L-PLANE FOR THE INPUT CONJUGATELY MATCHED CONDITIONALLY STABLE AMPLIFIER

Input Conjugately Matched Conditionally Stable Amplifier		
Γ_L-Plane	Constant μ-Contour	Constant μ'-Contour
Center	$\frac{\lvert C_2\rvert(1-\mu^2)}{E_2(1-\mu^2)-2\lvert T_{12}T_{21}\rvert(k-\mu)}\hat{c}_L$	$\frac{2\lvert C_2\rvert(k-\mu')}{\lvert T_{12}T_{21}\rvert[1-(\mu')^2]+2D_2(k-\mu')}\hat{c}_L$
Radius	$\left\lvert\frac{(T_{12}T_{21})[\mu^2-2k\mu+1]}{E_2(1-\mu^2)-2\lvert T_{12}T_{21}\rvert(k-\mu)}\right\rvert$	$\left\lvert\frac{(T_{12}T_{21})[(\mu')^2-2k\mu'+1]}{\lvert T_{12}T_{21}\rvert[1-(\mu')^2]+2D_2(k-\mu')}\right\rvert$
Operating Gain, G_P	$\frac{(1-\mu^2)}{2\mu(1-k\mu)}\cdot MSG$	$\frac{2(k-\mu')}{[1-(\mu')^2]}\cdot MSG$

the centers of the circles, i.e.,

$$\frac{|C_1|}{D_1+\dfrac{1}{g_a}}=\frac{|C_1|[1-(\mu')^2]}{E_1[1-(\mu')^2]-2|T_{12}T_{21}|(k-\mu')}.$$

Rearranging the above formula yields

$$g_a=\frac{[1-(\mu')^2]}{2|T_{12}T_{21}|\mu'(1-k\mu')}. \tag{14}$$

Or

$$G_A=\frac{|T_{21}|[1-(\mu')^2]}{2|T_{12}|\mu'(1-k\mu')}=\frac{[1-(\mu')^2]}{2\mu'(1-k\mu')}\cdot \text{MSG} \tag{15}$$

where MSG (maximum stable gain) is defined as $|T_{21}/T_{12}|$.

IV. CONSTANT μ-CONTOURS IN THE SOURCE PLANE

Since the output of the amplifier must be conjugately matched, Γ_L is determined by the $\boldsymbol{U}$ matrix, i.e., $\Gamma_L = U_{22}^*(\Gamma_S, \boldsymbol{T})$. Now that Γ_L is defined, the OMN is determined up to an arbitrary transmission line added to the output. Since μ is invariant to the transmission line it can be ignored. Similar to the previous case, μ can be determined by considering only the transistor combined with the OMN as illustrated in Fig. 5. The S matrix for the combined transistor and OMN is therefore

$$\boldsymbol{V}(\Gamma_S,\boldsymbol{T})=\frac{1}{1-T_{22}U_{22}^*}\cdot\begin{pmatrix} T_{11}-\Delta_{\boldsymbol{T}}U_{22}^* & T_{12}\sqrt{1-|U_{22}|^2}\\ T_{21}\sqrt{1-|U_{22}|^2} & T_{22}-U_{22}\end{pmatrix}. \tag{16}$$

The stability parameter μ of the amplifier is then

$$\mu=\frac{1-|V_{11}|^2}{|V_{22}-\Delta_{\boldsymbol{V}}V_{11}^*|+|V_{12}V_{21}|}.$$

Or, alternatively

$$\mu=\frac{|V_{22}-\Delta_{\boldsymbol{V}}V_{11}^*|-|V_{12}V_{21}|}{|V_{22}|^2-|\Delta_{\boldsymbol{V}}|^2}. \tag{17}$$

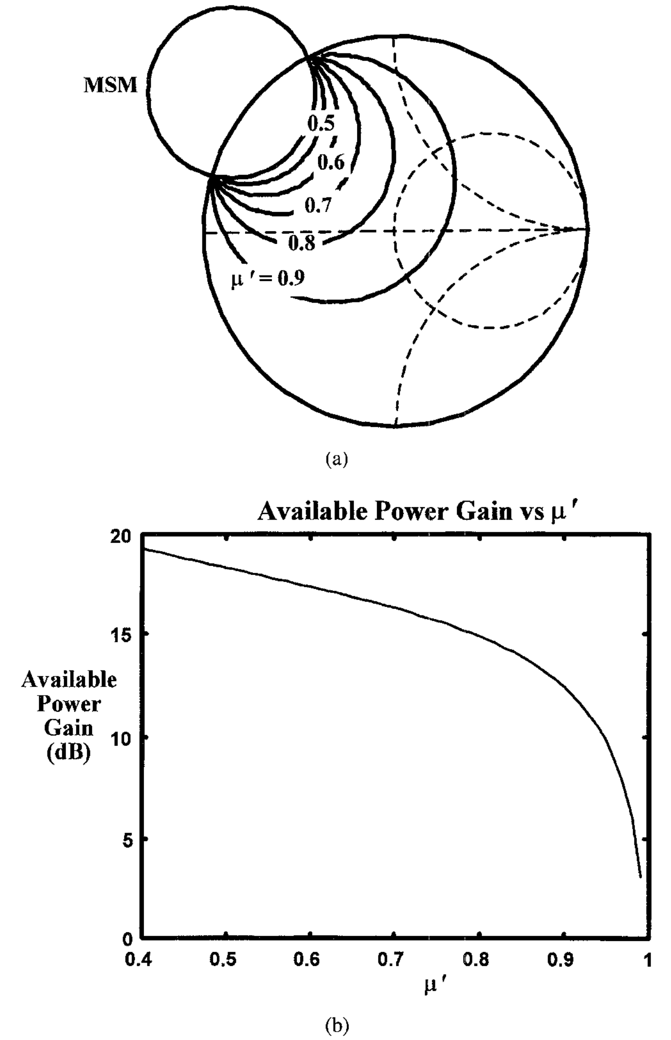

Fig. 6. (a) Γ_S plane showing MSM circle with constant μ'-contours and (b) available power gain versus μ'.

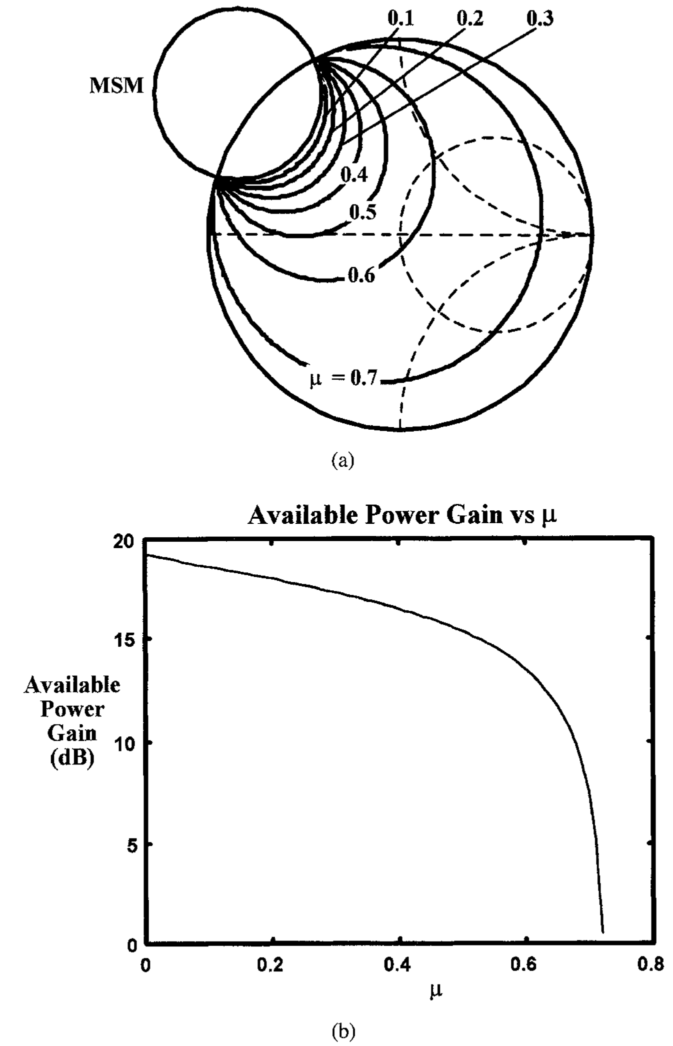

Fig. 7. (a) Γ_S plane showing MSM circle with constant μ-contours and (b) available power gain vs. μ.

Substituting the matrix elements from (16) yields

$$\mu = \frac{|C_2 - B_2U_{22} + C_2^*U_{22}^2| - |T_{12}T_{21}(1-|U_{22}|^2)|}{|T_{22}-U_{22}|^2 - |\Delta_{\boldsymbol{T}} - T_{11}U_{22}|^2}$$

Since $U_{22} = \Gamma_{\mathrm{OUT}}$

$$\mu = \frac{|C_2 - B_2\Gamma_{\mathrm{OUT}} + C_2^*\Gamma_{\mathrm{OUT}}^2| - |T_{12}T_{21}(1-|\Gamma_{\mathrm{OUT}}|^2)|}{|T_{22}-\Gamma_{\mathrm{OUT}}|^2 - |\Delta_{\boldsymbol{T}} - T_{11}\Gamma_{\mathrm{OUT}}|^2}. \tag{18}$$

Equation (18) is seen to be similar to expression (5) for μ' where the first term in the numerator can be expressed in terms of Γ_{OUT}^+ and Γ_{OUT}^- as below.

$$|C_2 - B_2\Gamma_{\mathrm{OUT}} + C_2^*\Gamma_{\mathrm{OUT}}^2| = |C_2(\Gamma_{\mathrm{OUT}} - \Gamma_{\mathrm{OUT}}^+)(\Gamma_S - \Gamma_{\mathrm{OUT}}^-)|$$

Γ_{OUT}^+ and Γ_{OUT}^- are the points where the output mapping circle intersects the USC. The invariant points $\Gamma_S^{\pm}$ in the source plane are mapped to the points $\Gamma_{\mathrm{OUT}}^{\pm}$ in the output plane via the bilinear transformation (2).

The term inside the second absolute sign in the numerator of (18) is positive provided that Γ_{OUT} remains in the USC, i.e., $|\Gamma_{\mathrm{OUT}}| < 1$, which is equivalent to saying that Γ_S is on the stable side of the source stability circle which is indeed the case.

Examination of (18) for μ in terms of Γ_{OUT}, and comparing it to (5) for μ' in terms of Γ_S, one sees that they are similarly formulated except that 1) Γ_{OUT} is not conjugated while Γ_S is, and 2) the subscripts 1 and 2 of matrix $\boldsymbol{T}$ related parameters are interchanged. This facilitate finding the solutions for constant μ contour on the Γ_{OUT} plane where they are known to be circles and can be represented by

$$\left|\Gamma_{\mathrm{OUT}} - \frac{|C_2|(1-\mu^2)}{E_2(1-\mu^2) - 2|T_{12}T_{21}|(k-\mu)}\hat{c}_0\right| = \left|\frac{(T_{12}T_{21})(\mu^2 - 2k\mu + 1)}{E_2(1-\mu^2) - 2|T_{12}T_{21}|(k-\mu)}\right| \tag{19}$$

where

$$\hat{c}_0 = \frac{C_2}{|C_2|} \tag{20}$$

$\hat{c}_0$ is the unit vector along the direction from the center of the USC to the center of the output mapping circle.

Substitution of (2) into the above equation yields the constant μ contour in the Γ_S plane, i.e.,

$$\left|\Gamma_S - \frac{2|C_1|(k-\mu)}{|T_{12}T_{21}|(1-\mu^2)+2D_1(k-\mu)}\hat{c}_S\right| = \left|\frac{(T_{12}T_{21})(\mu^2-2k\mu+1)}{|T_{12}T_{21}|(1-\mu^2)+2D_1(k-\mu)}\right| \quad (21)$$

Since the constant μ contour in the Γ_{OUT} plane passes through the $\Gamma_{OUT}^{\pm}$ invariant points, its bilinearly trasformed counterpart, i.e., the constant μ contour in the Γ_S plane, must also passes through the $\Gamma_S^{\pm}$ invariant points. Consequently, equating the center of the constant μ contour on the Γ_S plane with the center of the available gain circle, (13), yields the relationship between the available power gain G_A and the stability parameter μ of the amplifier, i.e.,

$$g_a = \frac{2(k-\mu)}{|T_{12}T_{21}|(1-\mu^2)}. \quad (22)$$

Or

$$G_A = \frac{2|T_{21}|(k-\mu)}{|T_{12}|(1-\mu^2)} = \frac{2(k-\mu)}{(1-\mu^2)}\cdot \text{MSG}. \quad (23)$$

V. Constant μ (or μ')-Contours in the Load Plane

The above development has concentrated on designing a conditionally stable amplifier ($0<k<1$) with the output conjugately matched, i.e., the design process started with Γ_S. Similarly, an input conjugately matched conditionally stable amplifier can be designed starting with Γ_L. In this case, the constant μ contour is

$$\left|\Gamma_L - \frac{|C_2|(1-\mu^2)}{E_2(1-\mu^2)-2|T_{12}T_{21}|(k-\mu)}\hat{c}_L\right| = \left|\frac{(T_{12}T_{21})[\mu^2-2k\mu+1]}{E_2(1-\mu^2)-2|T_{12}T_{21}|(k-\mu)}\right| \quad (24)$$

where $\hat{c}_L = C_2^*/|C_2|$ [2].

These contours correspond to constant operating power gain circles in the load plane. Equating the expressions for the centers of the constant μ and constant g_p circles yields the formula for the operating power gain vs. μ as follows.

$$g_p = \frac{(1-\mu^2)}{2|T_{12}T_{21}|\mu(1-k\mu)}. \quad (25)$$

Or

$$G_P = \frac{|T_{21}|(1-\mu^2)}{2|T_{12}|\mu(1-k\mu)} = \frac{(1-\mu^2)}{2\mu(1-k\mu)}\cdot \text{MSG}. \quad (26)$$

Likewise, constant μ' contours can also be found on the load plane to be circles in the following form:

$$\left|\Gamma_L - \frac{2|C_2|(k-\mu')}{|T_{12}T_{21}|[1-(\mu')^2]+2D_2(k-\mu')}\hat{c}_L\right| = \left|\frac{(T_{12}T_{21})[(\mu')^2-2k\mu'+1]}{|T_{12}T_{21}|[1-(\mu')^2]+2D_2(k-\mu')}\right|. \quad (27)$$

The formula for the corresponding operating power gain is

$$g_p = \frac{2(k-\mu')}{|T_{12}T_{21}|[1-(\mu')^2]}. \quad (28)$$

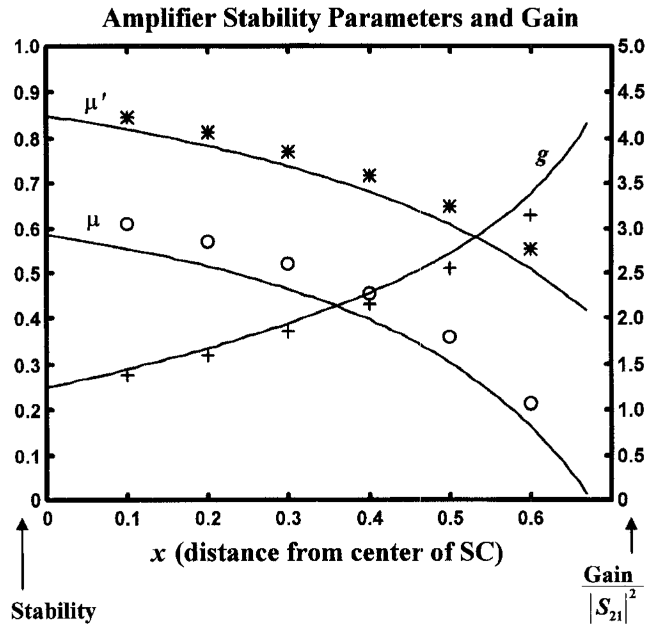

Fig. 8. Amplifier stability and gain as a function of x. Discrete data points show microstrips implementation of the design.

Or,

$$G_P = \frac{2|T_{21}|(k-\mu')}{|T_{12}|[1-(\mu')^2]} = \frac{2(k-\mu')}{[1-(\mu')^2]}\cdot \text{MSG}. \quad (29)$$

VI. Summary

The formulas for the constant μ and μ' contours and their associated gains as described in the previous two sections are summarized in Tables I and II. These formulas can be easily programmed in the existing CAD software packages by engineers interested in designing conditionally stable amplifiers.

VII. Design Example

The design procedure is illustrated for a design using a Mitsubishi transistor operating at 6 GHz. Figs. 6(a) and 7(a) show the source plane MSM circle and constant μ' and μ contours respectively. In both cases the μ' and μ contours are observed to also be available gain circles. The relationships of available gain vs. μ' and μ are shown in Figs. 6(b) and 7(b). Therefore, choosing the available gain determines both the μ' and μ stability factor that will result for the completed amplifier.

Since the constant μ contour are gain circles then it is sufficient to choose Γ_S a distance, x, from the Smith Chart center along the line of lowest gradient defined by the unit vector $\hat{c}_S$ which points to the center of the MSM circle. In this case μ, μ', and the available gain are functions of x and are plotted in Fig. 8.

Several input and output matching networks were designed for an externally biased amplifier using Mitsubishi MGF-4301A MESFET at 6 GHz. The designs assume a range of x values with IMN and OMN microstrip boards to be integrated on a carrier with a raised center ridge for transistor grounding. The amplifier layout is illustrated in Fig. 9.

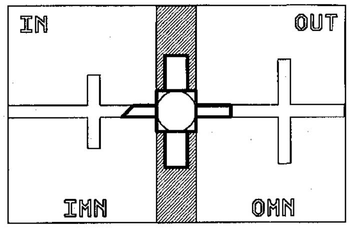

Fig. 9. Amplifier layout for $x = 0.4$.

The resulting S-parameters for each amplifier were then used to calculate μ, μ' and normalized gain. These data points have been added to the plot in Fig. 8 for comparison. The fact that the gain is less than the ideal and the stability parameters are greater than the ideal is attributable to the losses in the dielectric and microstrip lines.

VIII. Conclusion

A procedure has been developed for designing conditionally stable amplifiers where the input and output matching networks can be selected with an *a priori*, quantitative knowledge of the gain and the stability parameter, μ and μ'. This permits the designer to make a trade-off between gain and stability. Contours of constant μ (or μ') are shown to coincide with constant gain circles and explicit formulas for the center and radius are given. The relationship between gain μ and (or μ') is also given. The technique is illustrated in the design of a conditionally stable C band amplifier.

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